Turbulent Heat and Mass Transfer from Rough Surfaces

G. A. HUGHMARK

Ethyl Corporation
Baton Rouge, Louisiana

A recent analysis of heat and mass transfer presents equations for the wall region and core with turbulent flow in smooth circular pipes (*1*). The wall region equation for N_{Sc} or $N_{Pr} > 1$ is

$$k = 0.0816\ u^{*}\ N_{Sc}^{-2/3} \quad (1)$$

and the core equation represents the sum of the molecular and eddy contributions

$$k_c = k_m + k_{EC} \quad (2)$$

Equation (1) represents a developing boundary layer model for the wall region. Rough surface data can be used for a comparison of the wall region response for smooth and rough surfaces.

ROUGH SURFACE CIRCULAR PIPE

Equations (1) and (2) can be written in heat transfer terms

$$\frac{h_w}{\rho C_P} = 0.0816\ u\sqrt{f/2}\ N_{Pr}^{-2/3} \quad (3)$$

and

$$h_c = h_m + h_{EC} \quad (4)$$

For fully developed turbulence and $N_{Pr} > 1$, $h_{EC} >> h_m$ so $h_c \approx h_{EC}$. Again in heat transfer terms

$$h_c = h_{EC} = u\sqrt{f/2}\ k_{EC}^{+}\ \rho\ C_P \quad (5)$$

With series resistance of the wall region and core

$$\frac{1}{h} = \frac{1}{h_w} + \frac{1}{h_c}$$

Substitution of Equations (3) and (5) for the wall region and core yields for smooth pipes

$$\frac{N_{Pr}^{2/3}}{\dfrac{\sqrt{f/2}}{N_{St}} - \dfrac{1}{k_{EC}^{+}}} = 0.0816 = \alpha \quad (6)$$

Gowen and Smith (*2*) present heat transfer data for water and 30% ethylene glycol in water for several rough surfaces. The rough surfaces were obtained by soldering a 12 mesh screen to the wall and by soldering 20-25 mesh balls 0.125 in. apart on the wall surface. These data can be analyzed with Equation (6) and the k_{EC}^{+} correlation from the prior paper as an estimate of the coefficient for Equation (1). Figure 1 shows the data. The coefficient of 0.0816 for smooth pipe appears to fit the rough pipe data in the Reynolds number region of 6000 to 15000. This indicates that the boundary layer analysis for the wall region with smooth pipe also applies to rough surfaces in this Reynolds number range. The coefficient decreases with Reynolds number for $N_{Re} > 15000$. Figure 1 also shows the data of Kolar (*3*) for water in a 26 mm diameter tube with 0.5 mm deep threads.

FIXED BEDS—SPHERICAL PARTICLES

Carman (*4*) has shown that pressure loss for flow in a fixed bed of particles can be represented by flow in a tortuous channel.

$$\frac{\Delta P}{L_e} = \frac{2.5\ f\ u_e^2\ \rho}{D_e} \quad (7)$$

with

$$u_e = \frac{u}{\epsilon}\ \frac{L_e}{L}$$

Carman shows that $L_e/L = \sqrt{2}$ and that this results in $f = 16/N_{Re}$ at low Reynolds numbers $N_{Re} = D_e u_e/\nu$. This analysis can be applied to beds of spherical particles with the equation

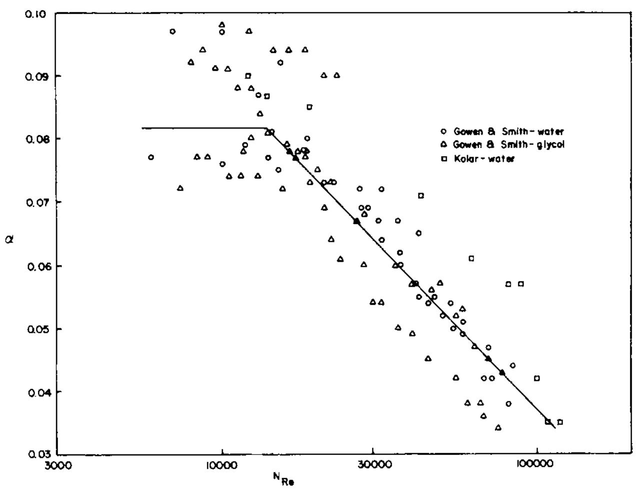

Fig. 1. Rough surface heat transfer.

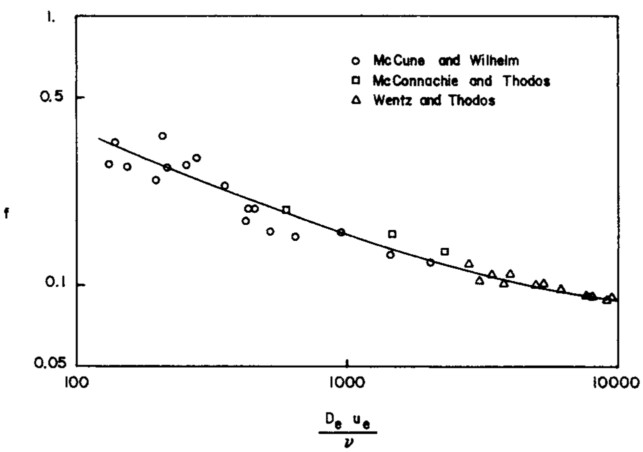

Fig. 2. Fixed bed friction factor.

$$D_e = 0.667 \frac{\epsilon}{1-\epsilon} D_P \qquad (8)$$

and extended to turbulent flow conditions in the bed. Figure 2 shows the friction factor-Reynolds number relationship obtained with equation (7) from the experimental data of McCune and Wilhelm (5), McConnachie and Thodos (6), and Wentz and Thodos (7). The heat and mass transfer analysis for pipe flow can also be applied to the fixed bed for channel flow. Equation (6) is then applicable but k_{EC}^+ is not known. An approximation can be made by assuming that k_{EC}^+ is a very large value because of the turbulence level in a fixed bed. The heat and mass transfer equations by analogy with smooth pipe flow then become

$$\frac{\dfrac{N_{Pr}^{2/3}}{\sqrt{f/2}}}{h/(\rho\ C_P\ u_e)} = \frac{\dfrac{N_{Sc}^{2/3}}{\sqrt{f/2}}}{k/u_e} = 0.0816 = \alpha \qquad (8)$$

Figure 3 shows the experimental data of McCune and Wilhelm for mass transfer of 2-napthol particles with water, McConnachie and Thodos, and Gupta and Thodos (8) for heat and mass transfer with air. The results indicate a coefficient of the same order of magnitude as that for smooth pipe in the Reynolds number range 200 to 600 and then a decreasing coefficient with increasing Reynolds number. This response is similar to that for rough surfaces and indicates that the heat and mass transfer analysis for a packed bed may be consistent with the rough surface channel model.

The rough surface analysis indicates that the wall region conditions are similar for rough and smooth pipe for the initial region of fully developed turbulence. At higher Reynolds numbers, the transfer rate appears to be less than the comparable smooth surface rate. This indicates that flow separation may occur at the rough surface so that a different turbulence structure may exist in the wall region. The correlations are not intended as design methods.

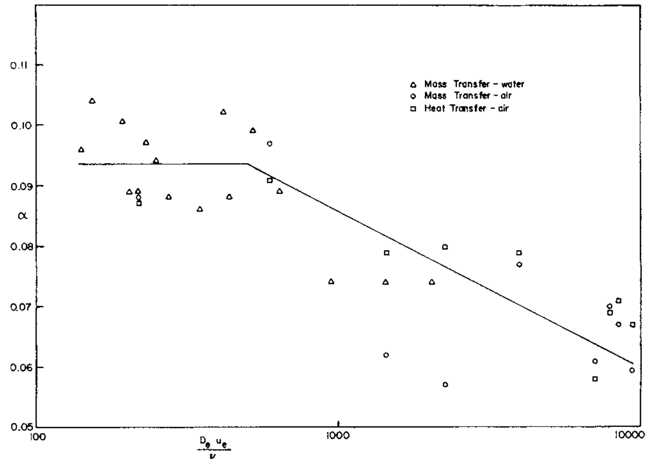

Fig. 3. Fixed bed heat and mass transfer.

NOTATION

C_P = specific heat
D_e = equivalent diameter of channel
D_P = particle diameter
f = Fanning friction factor
h = heat transfer coefficient
k = mass transfer coefficient
k^+ = k/u^*
L = bed depth
L_e = actual length of path taken by fluid in traversing depth L of bed
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
N_{Sc} = Schmidt number
N_{St} = Stanton number
ΔP = pressure difference
u = superficial fluid velocity
u_e = actual velocity in packing channels
u^* = shear velocity

Greek Letters

ϵ = packing void fraction
ν = kinematic viscosity
ρ = fluid density

Subscripts

c = core
EC = eddy diffusion, core
m = molecular
w = wall

LITERATURE CITED

1. Hughmark, G. A., *AIChE J.*, **17**, 902 (1971).
2. Gowen, R. A., and J. W. Smith, *Intern. J. Heat Mass Transfer*, **11**, 1657 (1968).
3. Kolar, V., *ibid.*, **8**, 639 (1965).
4. Carman, P. C., *Trans. Inst. Chem. Engrs. (London)*, **15**, 150 (1937).
5. McCune, L. K., and R. H. Wilhelm, *Ind. Eng. Chem.*, **41**, 1124 (1949).
6. McConnachie, J. T. L., and George Thodos, *AIChE J.*, **9**, 60 (1963).
7. Wentz, C. A., and George Thodos, *ibid.*, 358 (1963).
8. Gupta, A. S., and George Thodos, *Ind. Eng. Chem. Fundamentals*, **3**, 218 (1964).